Two Equations to the Stars - Part One: The Rocket Equation

How to explain rocket maths to mid-secondary school students

John I Davies

There are two currently feasible ways to propel a space probe to the stars, fusion rockets and laser sails - each governed by a simple equation.

Here John Davies explains the first of these equations and how middle secondary school students can be introduced to the mathematics, physics and engineering behind all rockets, including the ones that can propel us to the stars. In a separate Addendum on our website, Atholl Hay explains how teachers and advanced students can derive the equation from Newton's second law using the steps as used numerically in this article. Part Two, in the next issue of Principium, will explain the simple equation for laser and solar sail propulsion.

Who is this article for? This article is primarily intended for teachers introducing school students to rocket physics, maths and engineering in mid-secondary school (middle school in USA). It avoids using calculus (integration) or momentum since these concepts are unlikely to have been introduced to students at this level - and many will never cover them. The objective is to give these students an accessible yet fairly rigorous understanding of the fundamentals of rocket propulsion. It uses a numerical approximation in a spreadsheet - thus avoiding those unfamiliar topics while introducing students to the consequences of the equation, the beginnings of calculus and the idea of numerical approximation as used elsewhere in physics, engineering and beyond.

Reading order

• **Teachers** should read the *1-Introduction*, *2-Background and Approach* (2.1 and 2.2) - about 550 words. If the approach is interesting then read the rest of the article. If it is not interesting then perhaps tell i4is why (john.davies@i4is.org) so we can refine this. This article is accompanied by a derivation of the Tsiolkovsky equation using similar steps to the spreadsheet, RocketCalcSimpleForTsiolkovsky [1]. The derivation is in an Addendum [2] published alongside this issue.

• **School students** should read *1-Introduction* and *3-For School Students - What you can achieve*. If they feel confident then read *4-For School Students - The Spreadsheet* - as far as possible and then report progress to your maths, science or engineering teacher and ask them to take a look at this article. If you don't feel confident then ask a teacher to help with the maths, science or engineering.

[1] RocketCalcSimpleForTsiolkovsky https://i4is.org/wp-content/uploads/2022/03/RocketCalcSimpleForTsiolkovsky.xls
[2] *The Tsiolkovsky Rocket Equation - A Parallel Derivation*, Atholl Hay
https://i4is.org/wp-content/uploads/2022/04/The-Rocket-Equation-Principium_Addendum_7-converted.pdf

▶

Some heroes -
Al-Karismi,
Newton and
Tsiolkovsky

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The work of a mathematician, Al-Khwarizmi (780-850 CE) - bottom right, a physicist, Newton (1642-1726 CE) bottom centre, and a maths teacher and engineer, Tsiolkovsky (1857-1935 CE) - top - gave us the way to understand what rockets can do

1 Introduction

This is the first of two articles for Principium explaining how to understand the equations which define our ability to reach both our Solar System and the stars. Our aim in this article is to provide a way to understand the Tsiolkovsky rocket equation without using mathematics beyond basic secondary school level. It is intended for teachers, self-motivated secondary school students and interested people in general. More about its objectives in section *3 - For School Students - What you can achieve*.

The second article will explain the Robert Forward's laser sail equation - which governs the propulsion challenges of low mass probes such as i4is Project Dragonfly (2014) and the ongoing Project Starshot financed by Breakthrough Initiatives.

2 Background and Approach

This article follows from a presentation given to a number of UK secondary schools over recent years. This section is based on the material used for these presentations.

2.1 The Story

Our story begins with Al-Khwarizmi, Isaac Newton and Konstantin Tsiolkovsky, three heroes of Maths, Physics and Engineering. Al-Khwarizmi was the mathematician who taught us how to do algebraic manipulation. For example if $x = y/z$ then it follows that $xz = y$ - just multiply both sides of the first equation by z. Your multiplication is $xz = (y/z)z$ but $z/z = 1$ so right hand side is just y.
Isaac Newton gave us his three laws of motion and the algebraic one is the second law. $F = ma$, Force = Mass times Acceleration [1]. Al-Khwarizmi tells us this must mean that $a = F/m$. Konstantin Tsiolkovsky applied $a = F/m$ to a rocket where the complication is that the rocket gets lighter as it uses up fuel and gets lighter, so M keeps decreasing and acceleration (a) therefore gets bigger.

[1] Strictly speaking F is the *unbalanced* force or net force. Where multiple forces act then only the sum of these forces produces the acceleration.

Tsiolkovsky wanted to know how fast the rocket flies after all the fuel is used so he used integration on a= F/m to add all the accelerations as the rocket uses up the fuel and got deltaV = Ve ln(Mo/Mf), which looks like a cousin to a = F/m.

His equation says -

Change in velocity[deltaV] = rocket Exhaust Velocity[Ve] multiplied by Natural Logarithm[ln] of initial mass of the rocket[Mo] divided by final mass of the rocket[Mf] when all the fuel is used [1]. This equation -

deltaV = Ve ln(Mo/Mf)   [2]

- governs all rockets, in fact all reaction-propelled things from the mighty Saturn 5 which took humanity to the Moon to the tiny squid which can squirt water to propel themselves and even let them fly above the water for short distances.

It's important to remember that no rocket vehicle is immune from other forces such as gravitation and friction.

2.2 The Problem

But school students don't learn how to do integration until they get towards the end of school mathematics - and many never do it at all. And most derivations of the equation also use the concept of momentum which is also introduced late in school physics.

So how can school students get their heads around the rocket equation? The answer here is to use simple numerical methods - in fact a spreadsheet [3]. Here's how this works -

2.3 The Steps

First we need Newton's third, most "commonsensical", law - every action has an equal and opposite reaction. So if you try to push something it pushes you back, try it on ice where friction does not complicate things much. Here's an example with a skateboard, a man and a big ball -

But how do you "push" a rocket?

Like this guy on a skateboard

He throws the ball that way

He goes that way

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**Newton's third law for a skateboard, a man and a big ball**

You can also understand this using the idea of conservation of momentum - but this is only introduced late in UK secondary schools.

[1] Older textbooks will use the notation $\log_e$ for the natural logarithm.
[2] using capital M and V here to make the subscripts clearer.
[3] In UK schools spreadsheet work was standard up to a few years ago but some questionable political decisions have made it less common. Nevertheless in many UK schools and most schools elsewhere some facility with spreadsheets is still taught and spreadsheet skills are useful in life outside schools so it's easy to justify introducing it everywhere that basic computer skills are taught. School PCs will normally have a spreadsheet program installed, Excel or LibreOffice Calc.

◀ And like the guy on the skateboard -

It goes that way

Rocket throws the stuff that way

APOLLO 11
Off to the Moon
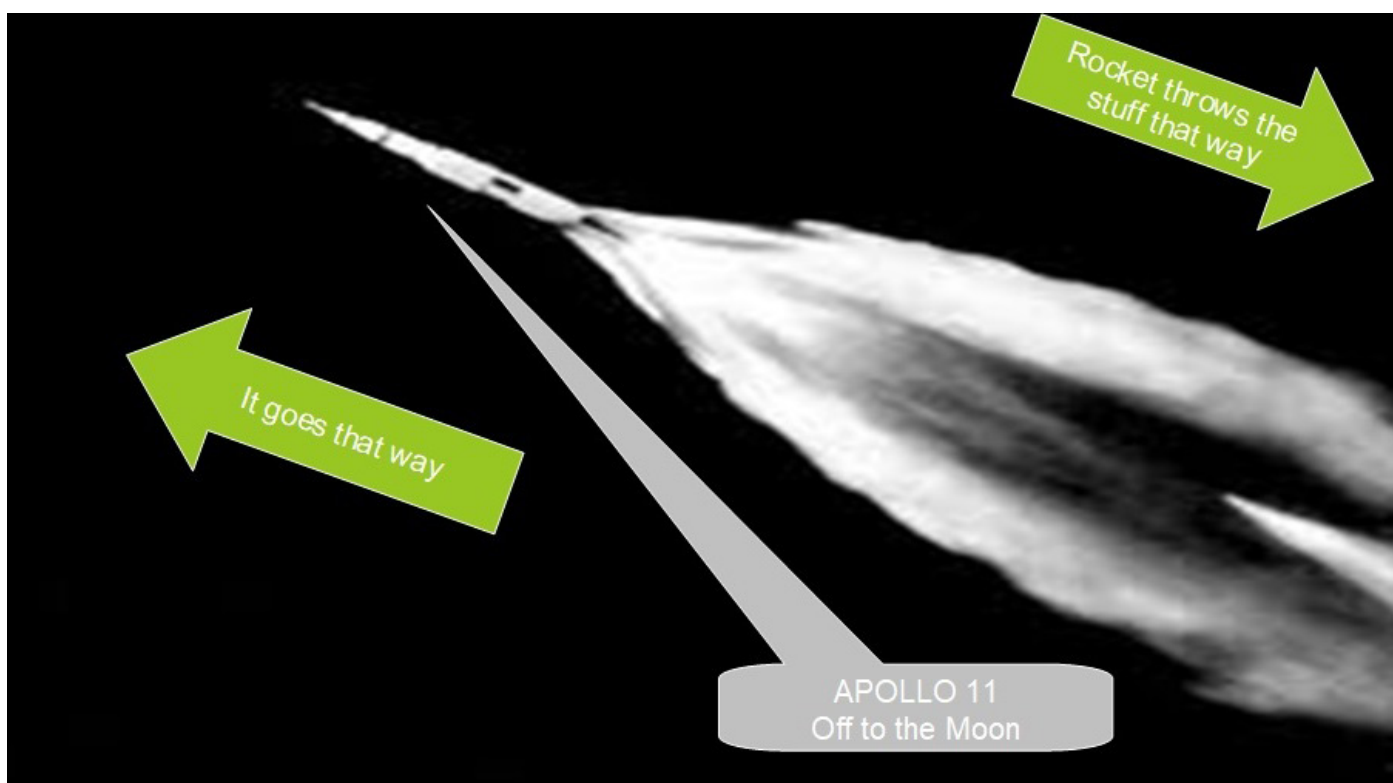
Newton's third law on a bigger scale - Saturn 5 lifting Apollo 11.

The force here is 34,000 kN but the mass keeps reducing as the fuel is burnt and pushed out of the back. So the initial acceleration is about 34,000,000 Newtons/2,965,000 kg= 11.5 metres per second per second minus gravity which is 9.8 metres per second per second. So the whole thing only accelerates at 11.5-9.8=1.7 metres per second per second - and it looks quite slow if you watch the video. But as the mass reduces the acceleration increases.

Newton's second law is - F = ma, force = mass times acceleration.

Al-Khwarizmi tells us this must mean that a = F/m, acceleration = force divided by mass.

Or "the harder you push, the greater the acceleration"!

As the Apollo 11 Launch video shows, big heavy rockets start slowly! And gravitation is the other powerful force operating here.

How do we convert rocket acceleration -
a = F/m
- into the velocity we get by using all the fuel?

We have to convert acceleration into velocity. We can assume that the F from the rocket thrust is constant but the mass we are accelerating keeps reducing as the fuel is burnt and expelled as exhaust.

But we know the total mass of the rocket with all its fuel and the mass of the fuel so we know the initial and final mass of the whole thing. We also know that F, the force produced by the rocket thrust, from F = ma. In this equation the mass (m) is the exhausted fuel and the acceleration of the exhausted fuel, acceleration (a), is constant because the rate of use of the fuel is constant.
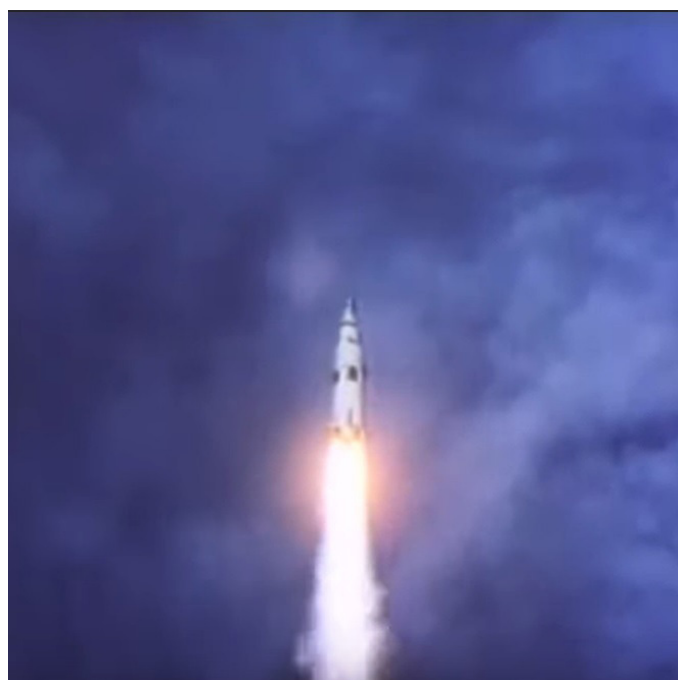
Apollo 11 Launch video: www.youtube.com/watch?v=ygBxN5Ui0aM

◀ If you know calculus you can integrate the equation - ∫a = ∫F/m

The integration limits are time interval so ∫a becomes the change in velocity, Δv, no integration required. The right hand side is more tricky. Tsiolkovsky did the integration and came up with his famous equation -

$$\Delta V = Ve \ln\left(\frac{Mo}{Mf}\right)$$

again using capital M and V to make the subscripts clearer

So the ΔV the rocket provides is the exhaust velocity, Ve, multiplied by the natural log (ln) of the mass ratio - the initial mass (Mo) divided by the final mass (Mf).

But how can we demonstrate that this works to the school students who haven't "done" integration and have not been introduced to the concept of momentum, as used in conventional derivations of the equation?

We have two tasks -

Convince students that the equation works while avoiding both momentum and integration.

Convince teachers that the method we use will allow the equation to be derived using only integration - avoiding the concept of momentum.

The sections **For School Students - The Spreadsheet** and **For Teachers - Derivation - Paralleling the Spreadsheet** below address these two tasks.

3 For School Students - What you can achieve

To understand how the physics of rockets lies behind the engineering which designs them you would normally need to understand both the concept of momentum and the mathematical operation called integration. Neither of these is required by the approach we suggest in this article.

So if you want to understand the basics of rockets you only need some very basic algebra and how to use a simple spreadsheet on your computer.

We hope that the next section will give you the tools to do this and enable you to -

Work out how good any sort of rocket is - to achieve anything from moving or rotating your small satellite as it orbits the Earth to getting your starship to the star you want it to go to!

Understand the beginnings of calculus - which you will find useful to do things in subjects

including rocket engineering, petrol and diesel engine design, construction engineering, accountancy, economics and just about all aspects of physics.

And, we hope, give you an idea of the power of mathematics and the satisfaction of understanding maths as a beautiful thing in itself and a pride in your membership of the human race which has accomplished such mighty things.

4 For School Students - The Spreadsheet

To understand rockets we need the work of Tsiolkovsky and two who came before him, Newton and Al-Khwarizmi. Here's how to use what they gave us.

4.1 Newton

Isaac Newton's three laws, in case you have not done them, are very simple but very powerful (I guess you have already heard of Newton ?).

The laws are -

#1 - an object will not change its motion unless a force acts on it;

#2 - the force on an object is equal to its mass times its acceleration;

#3 - when two objects interact, they apply forces to each other of equal magnitude and opposite directions.

In simple terms -

#1 - things don't move unless you push them;

#2 - the heavier (more massive) an object is, the more you have to push it, and the more you push the greater the acceleration you get;

#3 - when you push something you feel its pushing back on you.

The second law, #2, is the handy one for doing calculations. Mathematicians write is as -

F = ma

Force equals Mass times Acceleration.

On your computer it's -

F = m*a - computers are very literal; you have to tell them to multiply!

Schools will often use the idea of "balance of forces", adding and subtracting them to find out the net force. The methods in this article take one force at a time to keep things simple.

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4.2 Al-Khwarizmi

So Newton has given us F = ma but we need to find 'a', the rocket acceleration. Changing the subject to 'a' will need algebra, the gift to us from Al-Khwarizmi [1]. A simple step leads to a = F/m. This is the equation we will use to calculate the acceleration of the rocket once we know the thrust F.

See the sidebar below, Step by Step Algebra, for a simple introduction to the algebra here.

Step-by-step algebra

So F=ma but if our rocket is m kilograms and we push it with F newtons of force then how much will it accelerate?

a=?

Here's how to understand this if you have not yet done algebra in maths. We'll use "*" as our multiply operator, as we do in computing.

You know that 3*6=18, what happens if we divide the left and right of this by 3 -

3*6/3=18/3

We don't really need to do the division on the left. We know that 3/3=1 and 6*1=6 so the left becomes just 6

On the right we started with just 18 and divided this by 3 too - which is also gives us 6.

Surprise surprise: 6=6!

So here's some simple algebra doing the same sort of thing -

x*y=z

What happens if we divide both sides by y?

x*y/y=z/y

But we know 3/3=1 and that anything divided by itself gives 1.

And anything divided (or multiplied) by one is just itself - unchanged.

So if x*y/y=z/y then x*1=z/y and so x=z/y

We have converted x*y=z into x=z/y

So let's take Newton's second law in computer form -

F=m*a

- and divide both sides by m -

F/m=m*a/m

Remember that m/m=1, so the right hand side will be -

F/m=a

- and so -

a=F/m

- which is what we need to calculate accelaration.

Not so clever? But doing maths without numbers is a big conceptual jump, it's algebra, and Al-Khwarizmi [1] made that jump. He was a mathematician who lived over 1000 years ago.

4.3 Tsiolkovsky

You have probably heard of Newton and of algebra (though maybe not Al-Khwarizmi) but to understand rockets you need someone who wants to make things work, an engineer. More about this in the note ***What is a Rocket Scientist?*** at the end of this article.

You have probably never heard of Konstantin Eduardovitch Tsiolkovsky but he was a maths teacher who thought hard about the engineering problem of getting into space. His equation uses Newton and Al-Khwarizmi to solve the problem. He showed that it would be hard but not impossible. He died in 1935 so he never saw the results of his work - satellites, Mars probes, Moon missions and even probes beyond the Solar System.

Newton and Al-Khwarizmi give us a way to calculate the acceleration of a rocket -

a = F/m

- but how much speed do we get out of a rocket when the mass, m, reduces - by being expelled to get the force you need? Tsiolkovsky wanted to know how fast the rocket would fly once all the fuel was used up. He added all the accelerations on the left hand side and used integration and the concept of momentum on the right hand side to get his famous rocket equation -

$$\Delta V = V_e \ln\left(\frac{M_o}{M_f} \right)$$

- multiplication is implied here - you would need to put a "*" after Ve to keep your computer happy.

ΔV (pronounced delta-vee) [2] is the new speed of the rocket after it burns out. Ve is the exhaust velocity - how fast the rocket exhaust comes out of the rocket. The tricky problem is that the mass of the rocket keeps reducing as the fuel is used up. That's why when you see a big rocket launch it seems to start quite slowly. Tsiolkovsky's integration produces a function called the natural log (ln). He uses that function on the mass ratio of the rocket. That's the initial mass (Mo) divided by the final mass (Mf).

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[1] Muhammad ibn Musa al-Khwarizmi en.wikipedia.org/wiki/Muhammad_ibn_Musa_al-Khwarizmi.
[2] The Greek letter Δ "delta" is used in maths, physics and engineering to mean "change in". So Δv is change in velocity.

4.4 How do we avoid integration and momentum?

Most school students don't do integration and momentum until late in secondary school (high school in the USA) - and lots don't do them at all. So can we find a way to do what Tsiolkovsky did without these two bits of maths?

How about just "adding the acceleration for each second"? So calculate the speed added each second and add the extra speed every second. This is called a "numerical approximation". And it's used a lot in engineering – and practical sciences like meteorology. We would not get good weather forecasts (maybe not in UK!) without numerical approximation on massive computers. But you don't need monsters like the UK Met Office supercomputer to approximate the rocket equation. A simple spreadsheet will do the job.

Here's how -

Start with the **Mass of rocket** - starting (for example) with 100 kg

Let's decide to recalculate every 10 seconds

If we use 0.1 kg of fuel per second then in 10 seconds the **Mass of exhaust** used will be 10 times that, 1kg

Choose the **Velocity of exhaust** - even a firework rocket can manage 1,000 metres per second.

We already decided to recalculate every 10 seconds so between each calculation the **Time** interval is 10 seconds

We'll use our spreadsheet to calculate the **Acceleration of exhaust** in metres per second per second (note that extra "per second" - acceleration is the rate of change of velocity)

Here's the headings and first row of the spreadsheet -

Mass of rocket	Mass of exhaust	Velocity of exhaust	Time interval	Acceleration of exhaust	Force produced	Acceleration produced	Added velocity of rocket	Velocity of rocket
100	0.00	1000.00	10.00	100.00	0.00	0.00	0.00	0

Just the first four columns have results because the **Acceleration of exhaust** hasn't done its 10 seconds worth of pushing yet.

From that we can fill in the second row of the spreadsheet and calculate the effect of the **Acceleration of exhaust** on the rocket...

◀ **Force produced** by the rocket using Newton's F = ma - mass (m) is the mass of fuel used [1].
Acceleration produced using the other way of writing Newton a = F/m but the mass (m) here is now the mass of the rocket, which we know is getting less and less as the fuel is used up.
Now it's simple to take that **Acceleration produced** and work out the **Added velocity of rocket** in the 10 second interval we are using. For the first row of the spreadsheet the **Velocity of rocket** is just the **Added velocity of rocket** since it's starting from zero.

Mass of rocket	Mass of exhaust	Velocity of exhaust	Time interval	Acceleration of exhaust	Force produced	Acceleration produced	Added velocity of rocket	Velocity of rocket
100	0.00	1000.00	10.00	100.00	0.00	0.00	0.00	0.00
100	1.00	1000.00	10.00	100.00	100.00	1.00	10.00	10.00

- now display the calculations (press ctrl and ` for Excel) to see what we have done - Here it is with the row numbers and column letters also shown-

	A	B	C	D	E	F	G	H	I
1	Mass of rocket	Mass of exhaust	Velocity of exhaust	Time interval	Acceleration of exhaust	Force produced	Acceleration produced	Added velocity of rocket	Velocity of rocket
2	100	0	1000	10	=C2/D2	=B2*E2	=F2/A2	=G2*D2	0
3	=A2-B2	1	=C2	=D2	=C3/D3	=B3*E3	=F3/A3	=G3*D3	=I2+H3

But now we add a row to calculate for the next 10 second interval with the **Mass of rocket** now reduced by the **Mass of exhaust** used up in the first row. Calculate the whole row again and notice that the **Force produced** is the same but, because the **Mass of rocket** is less the **Acceleration produced** and the **Added velocity of rocket** have increased. More rows...

Mass of rocket	Mass of exhaust	Velocity of exhaust	Time interval	Acceleration of exhaust	Force produced	Acceleration produced	Added velocity of rocket	Velocity of rocket
100	0.00	1000.00	10.00	100.00	0.00	0.00	0.00	0.00
100	1.00	1000.00	10.00	100.00	100.00	1.00	10.00	10.00
99	1.00	1000.00	10.00	100.00	100.00	1.01	10.10	20.10

The rocket is accelerating faster.

[1] **A Note on the SpreadSheet Time Interval**
Sharp-eyed teachers and more advanced students will notice that the value of acceleration of exhaust, and thus force, calculated here depends upon the time interval chosen. This looks arbitrary. However, the length of the time interval Δt chosen affects the accuracy of the computation performed by the spread-sheet **but not the validity of the model of rocket propulsion**. If a "long" time interval is chosen the acceleration $a_e = v_e / \Delta t$ is decreased in value, but the amount of mass accelerated during the interval which is $m_e = k \Delta t$ where k is the burn-rate is increased. Conversely, if a "short" time interval is chosen, then the acceleration increases but the amount of mass accelerated in the interval decreases. So what matters is not the value of the acceleration per se, but the multiplication product,

$$m_e a_e = (v_e / \Delta t) \, k \, \Delta t = k v_e$$

which is constant since both k and v_e are constant attributes of the rocket motor. $k v_e$ is the constant the "thrust force" which acts on the decreasing mass of the rocket plus fuel.

The length of the time-interval does affect the accuracy of the computation performed by the spreadsheet as the rocket's velocity is taken to be constant during this period, whereas in reality it is constantly increasing. In essence this is the old problem of approximating a curve by series of straight-line segments. The shorter the segments are the better the approximation, and the longer they are, the worse the approximation.

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◀ Keep adding rows and see the complete spreadsheet. Here's some sample rows -

| Mass of rocket	Mass of exhaust	Velocity of exhaust	Time interval	Acceleration of exhaust	Force produced	Acceleration produced	Added velocity of rocket	Velocity of rocket
100	0.00	1000.00	10.00	100.00	0.00	0.00	0.00	0.00
100	1.00	1000.00	10.00	100.00	100.00	1.00	10.00	10.00
99	1.00	1000.00	10.00	100.00	100.00	1.01	10.10	20.10
98	1.00	1000.00	10.00	100.00	100.00	1.02	10.20	30.31
97	1.00	1000.00	10.00	100.00	100.00	1.03	10.31	40.61
...								
50	1.00	1000.00	10.00	100.00	100.00	2.00	20.00	708.17
49	1.00	1000.00	10.00	100.00	100.00	2.04	20.41	728.58
48	1.00	1000.00	10.00	100.00	100.00	2.08	20.83	749.41
...								
22	1.00	1000.00	10.00	100.00	100.00	4.55	45.45	1542.02
21	1.00	1000.00	10.00	100.00	100.00	4.76	47.62	1589.64
20	1.00	1000.00	10.00	100.00	100.00	5.00	50.00	1639.64
...								
5	1.00	1000.00	10.00	100.00	100.00	20.00	200.00	3104.04
4	1.00	1000.00	10.00	100.00	100.00	25.00	250.00	3354.04
3	1.00	1000.00	10.00	100.00	100.00	33.33	333.33	3687.38
2	1.00	1000.00	10.00	100.00	100.00	50.00	500.00	4187.38
1	1.00	1000.00	10.00	100.00	100.00	100.00	100.00	4287
Mass of rocket	Mass of exhaust	Velocity of exhaust	Time interval	Acceleration of exhaust	Force produced	Acceleration produced	Added velocity of rocket	Velocity of rocket

The header row is repeated at the bottom for clarity. Look at how the last row has the same Acceleration of exhaust and Force produced as in all the other rows but the Acceleration produced and Added velocity of rocket are much greater in each row - just because the rocket is getting lighter as the fuel is used up. The spreadsheet is available on the i4is website at i4is.org/wp-content/uploads/2022/03/RocketCalcSimpleForTsiolkovsky.xls.

4.5 Comparing Tsiolkovsky with your spreadsheet

Here's what Tsiolkovsky taught us -

$$\Delta V = Ve \ln\left(\frac{Mo}{Mf}\right)$$

The ln function produces something called a logarithmic curve. If you plot it as a graph, here's the curve you get from your spreadsheet ⇨
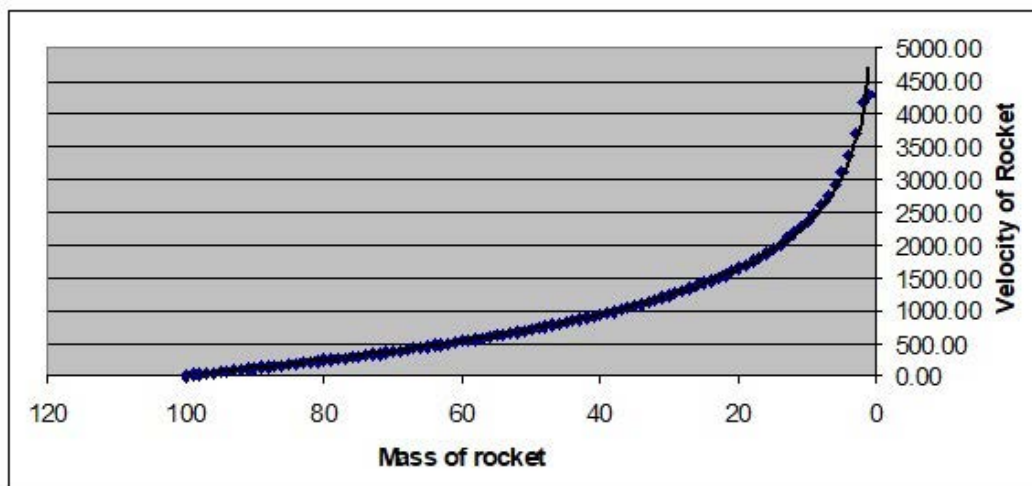
It looks a bit like an exponential curve but let us consult an "unbiased observer", Excel itself. Excel allows a trendline to be fitted to the curve and a function approximated. In this case Excel yields - y = -1015.1Ln(x) + 4683.3 - a natural log function displaced by the final value 4683.3. It's a logarithmic curve [1].
Let's take the same values you have in the spreadsheet and plug them in to -

$$\Delta V = Ve \ln\left(\frac{Mo}{Mf}\right)$$

Mo is 100 kg, Mf is 1 kg and Ve (the exhaust velocity) is 1,000 metres per second.
The mass ratio, the bit in the bracket, is 100/1 = 100. The natural log (ln) of 100 is 4.60 so the final (change in) velocity, ΔV, is 4.60*1000 = 4,600 metres per second.
This isn't exactly right but not a bad approximation for a spreadsheet! And you have a logarithmic curve just like the one produced by the Tsiolkovsky equation.

4.6 How to design better rockets

The Mf in the example, 1 kg, has to include the whole structure of the rocket and the cargo, called the payload, so it's a bit unrealistic! Imagine the rocket structure is 10 kg and the payload is 10 kg, total 20 kg. Looking at the 20 kg spreadsheet row -

Mass of rocket	Mass of exhaust	Velocity of exhaust	Time interval	Acceleration of exhaust	Force produced	Acceleration produced	Added velocity of rocket	Velocity of rocket
20	1	1000	10	100	100	5	50	1639.64

The velocity of the rocket is now 1639.64 metres per second but we are carrying a lot of structure we needed to hold the 80 kg of fuel we have used. Let's throw away that extra structure and start again with a 10 kg rocket, much lighter. This is the principle of the multi-stage rocket and you will notice that all real launches into space have more than one rocket stage.

[1] Mathematically a logaritmic curve is the inverse of an exponential curve. thirdspacelearning.com/gcse-maths/algebra/inverse-functions/

4.7 For Teachers - A better approximation and a two stage rocket

The spreadsheet approximation was first devised by John Davies of i4is but a better mathematician took a look and did produce an improved spreadsheet. Robert Freeland was the lead, with Michel Lamontagne, of the Firefly design - the most successful of the Project Icarus studies for probes propelled by a fusion rocket- extending the 1970s design of Project Daedalus of the British Interplanetary Society.

Robert saw potential in the original spreadsheet approximation and devised a better approximation with wider usefulness with -

- Two rocket stages
- More incremental steps in the spreadsheet for a better approximation
- Parallel use of the Tsiolkovsky equation to show the accuracy of the approximation

The Freeland spreadsheet is available on the i4is website at i4is.org/wp-content/uploads/2022/03/Incremental-Rocket-Example-Edited-by-Robert-Freeland.xls.

The columns in this version are -

Rocket Mass (tonnes), Exhaust Mass (tonnes), Exhaust Velocity (km/s), Time Interval (s), Acceleration of Exhaust (km/s2), Force Produced (MN), Acceleration Produced (km/s2), Added Velocity (km/s), Rocket Velocity (km/s), Elapsed Time (s), Tsiolkovsky Velocity (km/s)

4.8 More resources

There is a lot written about rocketry and the Tsiolkovsky equation. Here are just a few suggestions which may be useful background

The tyranny of the equation - a piece by NASA engineer/astronaut Don Pettit, almost cursing the limits set by Tsiolkovsky's equation [1] shows the frustration which Tsiolkovsky himself must have felt when he appreciated the consequences of his formula. As he said "Earth is the cradle of mankind but one cannot live in a cradle forever".

Other derivations of the equation - most use momentum and calculus -
NASA: www.grc.nasa.gov/WWW/K-12/rocket/rktpow.html
ESA: blogs.esa.int/rocketscience/2012/10/14/a-man-and-an-equation/
Wikipedia: en.wikipedia.org/wiki/Tsiolkovsky_rocket_equation#Derivation

Specific impulse is a measure of the efficiency of particular rocket technologies - both fuels and rocket motors. It is proportional to exhaust velocity [2].

Since it is measured in seconds it can be used instead of exhaust velocity to avoid numbers which differ between imperial and metric units. Exhaust velocity can, for example, be in feet per second or miles per hour versus metres per second and kilometres per hour but specific impulse is measured in seconds - which are the same in both systems of units.

Low mass/high exhaust velocity thrusters - ion propulsion gives high exhaust velocity and thus high specific impulse. So it uses less propellant to produce the same ΔV but ion thrusters are not scalable to the high thrust needed for launch from Earth [3].

4.9 Fission Rockets

Nuclear power has been advocated as a basis for rockets since at least the end of the Second World War in 1955. The difficulties of using a bulky and heavy device like the fission reactors used in power stations and ships has been much explored and solutions have been proposed that even go as far as suggesting an all-gas core. Recently interest has been revived as a way of speeding human Mars missions given the radiation load on astronauts inevitable with chemical rockets which would typically take about 6 months to get to Mars. More in a recent survey by NASA's Kurt A Polzin[4].

[1] The Tyranny of the Rocket Equation, www.nasa.gov/mission_pages/station/expeditions/expedition30/tryanny.html/
[2] Specific impulse: en.wikipedia.org/wiki/Specific_impulse
[3] NASA - Ion Propulsion, www.nasa.gov/centers/glenn/about/fs21grc.html
[4] Enabling Deep Space Science Missions with Nuclear Thermal Propulsion, assets.pubpub.org/gfaq1me4/01617915247833.pdf

◀ **4.9 Fusion Rockets**

Fusion power offers much greater promise but remains to be demonstrated. An article in Principium 22 August 2018 *Reaching the Stars in a Century using Fusion Propulsion* [1] introduces the propulsion technology of the most developed of the *Project Icarus* fusion-based probes.

5 Conclusion

In this article I hope I have introduced school students and their teachers to a way of understanding the maths and physics of rocket propulsion without requiring concepts in both subjects which students may not have encountered and which many will never encounter. My objective is to both demystify rockets and to provide an early inkling of those more advanced concepts. I would be very pleased to hear from both students and teachers with both questions and suggestions for improvement (john.davies@i4is.org).

The second article in this Principium series will explore the maths and physics behind laser sails and solar sails. Human ingenuity faced with the hard reality of the Tsiolkovsky rocket equation has, as usual, found a "work around" and Tsiolkovsky himself, all those years ago, also dreamed that light might propel future spacecraft.

Thanks to Atholl Hay and Graham Paterson (City of Glasgow College) for their advice. Errors and omissions remain my own.

6 Final Note: What is a rocket scientist?

In the biography by Michael Neufeld of Werner Von Braun, *Von Braun: Dreamer of Space, Engineer of War* [2], his biographer says -

> One term you will not find in this book is "rocket scientist." There has been a deep-rooted failure in the English-speaking media and popular culture to grapple with the distinction between science and engineering. Although the boundaries are fuzzy, and a leading historian of technology has argued that all we have now is a unitary "technoscience," I still find it useful to think of a spectrum. On one end is basic science, which aims at achieving an understanding of the laws of nature without regard for their practical application, and at the other is engineering, which is about creating technological devices to shape the world to human purposes. Although Werner von Braun got a doctorate in physics in 1934, he never worked a day in his life thereafter as a scientist. He was an engineer and a manager of engineers, and he used that vocabulary when he was talking to his professional peers. Thus the correct term is "rocket engineer."
>
> In short the term "rocket scientist" contains a contradiction. But the world recognises it and we, as practical engineers, must live with it.

The term scientist is also misapplied to a whole, and very important, subject area "Computer Science". Alan Turing is often called the first "computer scientist". To adapt from Neufeld "Alan Turing never worked a day in his life as a scientist" and neither do modern "computer scientists". The field includes engineering, both hardware and software, and mathematics - often referred to as "theoretical computer science". Turing was a mathematician of the first rank and an ingenious engineer of both hardware and software. He was also a significant contributor to philosophy, as his 1950 paper in the journal *Mind* shows [3]. He was never a scientist - of computers or anything else. But we must live with the contradiction here too.

Science discovers how the world (and the universe) works. Mathematics discovers how to manipulate symbols and numbers systematically. Engineering creates and enhances physical and abstract structures and mechanisms for specific human purposes - often using maths and science but sometimes by pure invention. ■

[1] *Reaching the Stars in a Century using Fusion Propulsion - A Review Paper based on the 'Firefly Icarus' Design,* Patrick J Mahon, i4is.org/reaching-the-stars-in-a-century-using-fusion-propulsion/
[2] Von Braun was project leader of the team which designed the Saturn 5 moon rocket and before that the V2 rockets which bombarded Antwerp and London in the Second World War.
[3] *Computing machinery and intelligence,* Alan M Turing 1950, *Mind*, 59, 433-460, www.cs.ox.ac.uk/activities/ieg/e-library/sources/t_article.pdf