

Cost-Optimal System Performance Maps for Laser-Accelerated Sailcraft

Adam Hibberd

Dr Kevin Parkin's systems engineering thinking has been fundamental to the work driven by Breakthrough Starshot. Our interstellar colleague Dr Al Jackson reviewed Dr Parkin's paper to the 2022 International Astronautical Congress 2022 in Principium 40, February 2023, page 24. Here Adam Hibberd takes a more extended look at this paper [1]. If we are to send probes to the nearest stars in the relatively near future then systems thinking must be an early and vital contributor to planning for this.

Dr Parkin's previous paper [2] assumed a 'point design' of 0.2c/1 gram and a precursor mission at 0.01c/7 mg.

The paper reviewed here covers a wide range of designs from 0.1 mg microbiome (a community of organisms) payloads to 100 kiloton payloads and from 0.0001c to 0.99c cruise velocities.

The main driver for what is possible in a design is its cost.

His previous paper analysed cost-optimal strategies using numerical techniques to model all the infrastructure (including the laser sail design) as well as the laser deployment and acceleration of the sail. This resulted in various issues, the main one being that the number-crunching to solve each problem was rather protracted.

Various developments since the construction of his previous paper have motivated him to develop alternative software which has improved convergence and a performance map over both payload mass and cruise velocity.

SYSTEM MODEL: This describes the propagation of a beam from a director at ground-level to a sailcraft, and that craft's resulting motion. Two objective functions are minimization of system capex (capital expenditure) and opex (operational expenditure).

Clearly the important factor for modelling the acceleration of a sailcraft is the light flux - that is the power per unit area - on its sail. Summed over the area of the sail we get the total power incident on the sail. Ideally all the light - and therefore power - generated by the beam director will impact on the sail. In practice there are various losses like for example due to spillage of the beam around the edges of the sail & attenuation due to Earth's atmosphere etc.

Imagine now a line extending in the direction from the beamer to the spacecraft (s/c) and beyond (and static wrt the beamer), with tick marks located at equidistant points along the way. What we have defined is the 'quasistatic' frame mentioned in the paper. The paper labels this direction 'z' and, as the laser beam accelerates the s/c, there will be a time lag between the power emitted by the beamer and that received by the s/c.

[1] IAC-22-D4.4.5 Cost-Optimal System Performance Maps for Laser-Accelerated Sailcraft, Kevin L G Parkin Parkin Research LLC, San Francisco, USA (arxiv.org/abs/2205.13138)

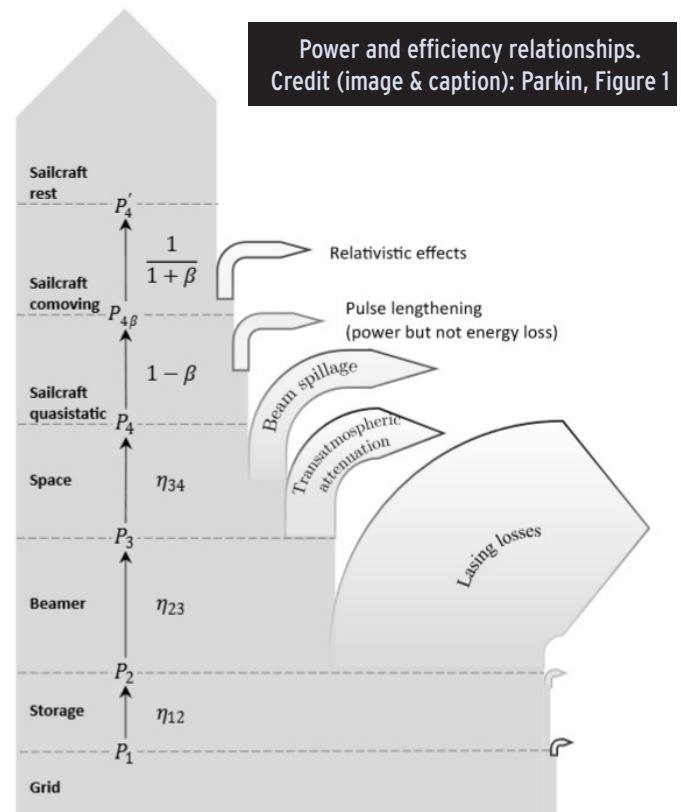
Now imagine the s/c is actually travelling at a velocity v somewhere along z , then we can define the co-moving frame as the instantaneous frame which is moving with the s/c at this velocity. The power received by the s/c in this frame is reduced by a factor $(1-v/c) = 1-\beta$. Furthermore Doppler shift (ie the increase in wavelength of light received by the s/c due to the receding velocity of the s/c) will introduce a further power loss in the form of $(1/(1+\beta))$.

Of course with the s/c travelling so quickly, we need to take into account relativistic effects in a relativistic rest frame at rest with the s/c. When this is all implemented, we get the overall factor $(1-\beta)/(1+\beta)$, which happens to be the relationship between power in a relativistic frame compared to a stationary frame, as derived by Einstein himself. Next the optics need to be modelled, more specifically the fractional transmission of beamer power to the s/c over the course of distance z . There are various models and they vary in the validity of their assumptions, and the degree of fudge which can be tolerated. Suffice to say a model which contains around 3% fudge is chosen for ease of analytical computation.

Now we can address the equations of motion, which require the force delivered to the s/c, and the paper makes the point that the force exerted is independent of reference frame, but is dependent on the power (after the respective losses mentioned above which change with z). Also the material for the sail is important in the form of its reflectance R and absorptance A . The equations can be derived by implementing the rate of change of momentum which of course is dependent on the rate of change of mass (through its relativistic connection with speed) as well as the rate of change of speed itself. It should be noted here that speed is not used directly but instead its ratio with the speed of light, β , and the well-known Lorentz Factor $\gamma = 1/\sqrt{1-\beta^2}$ [1].

Anyone with any experience of spacecraft trajectories will know that the equations of motion, which are usually in the form of an equation for the second derivative of position over time (in this case d^2z/dt^2) need to be integrated wrt time to get first velocity and then, when integrated again, position. However the paper asserts that the particular form of the equations lend themselves better to integrating the first derivative of velocity (or β actually) wrt position (so $d\beta/dz$), which will provide β as a function of position, z .

[1] As has been previously stated, the power incident on the s/c is essentially dependent on z only and thus an integration scheme based on the β as a function of z would seem more conducive to analysis than as a function of time.



First however to do this, simplifications need to be made, particularly that of assuming the power incident on the s/c is actually constant (rather than changing with z), since any amplification may induce temperatures which cannot be tolerated by the sail material - the so-called temperature-limited regime. Optionally, the power of the beam itself may be limited, due to physical constraints on the laser infrastructure, which gives us the alternative power-limited regime. Either of these simplifications yield closed solutions for the integrand of $d\beta/dz$, in turn giving β as a function of z .

Next we address the total energy required from the beamer and straightforwardly enough, it is simply the integral of power over time (as power is obviously rate of change of energy). It so happens that the equations yet again transform readily from a function of time to one of velocity (again as β actually), allowing a numerical integration to be conducted. It is this energy, Q , which must be minimized of course to minimise cost.

The system model can now be actuated via an algorithm explicitly defined in the paper. It amounts to using the beamer power-limited regime when the s/c's temperature so permits (ie is safely below an upper limit); but the temperature-limited regime when the s/c's temperature reaches a maximum

◀ limit, the former is a constant whereas for the latter the power incident on the s/c is constant. Now fully equipped with a system model in closed form it can be compared with that of Parkin's previous system model, and reassuringly trajectory curves are similar, though do differ in line with the fudge factor of 3% already highlighted above. Next the cost. There are two broad costs which need to be expressed algebraically, firstly CAPEX, which is the prior outlay needed for a particular beamer/lightsail combination. This is expressed as a linear function of three basic parameters, the total energy stored Q_2 , the peak power taken in real-time from the grid, P_3 (not factored in previous papers) and lightsail area A_3 . Secondly the OPEX cost, which is the cost of firing the beam, is proportional to total radiated energy, Q_1 . So what are the results? Essentially the most crucial is that laser power storage dominates the CAPEX and that increasing grid power provision reduces the CAPEX for lasersail masses up to 10 grams. Grid capability is limited by power generation capacity and the maximum total energy which can be drawn from the lines. Three point designs are outlined in three separate tables to analyse their respective merits. So what conclusions can be drawn from all this detailed analysis? Firstly the closed form solutions of the system model which have been derived has resulted in useful reduction in the complexity of the software used by the author as well as allowing for

a dramatic reduction in optimization time (1-2 orders of magnitude faster). Furthermore the results are valid, if one bears in mind the aforementioned fudge factors adopted. The new development enables investigation of entire performance maps over a huge range of parameters, as opposed to the old model which was confined to determining point designs. As far as precursor interstellar and solar system missions are concerned, where the requirements are high mass and low speed, inclusion of power drawn straight from the grid allows a reduction of costs of typically 1-3 orders of magnitude. A case in point is provided of a mass of 10 kg beamed to a speed of 0.001c (63 au/yr), with a destination of say Neptune. This would need a CAPEX of \$610M, way lower than the previous \$26B, where all laser power was drawn from storage. Smaller and larger missions were also analysed. A disadvantage of the former is the long time to accelerate the laser sail taking hours to days. If one looks at the future potential of larger spacecraft missions with say 7.4km 100kt vessel accelerated to 0.07c, that would also require long beam durations of around 20 days. Further ahead 380 PW peak radiated power can be envisioned and can also be simulated by the model derived by Parkin, but such missions would need power levels of twice that incident upon the Earth from the Sun, so new power sources would need to be developed eg nuclear fusion or space solar power.

Table 1: System model constants

1.06 μm wavelength
60 000 km initial sail displacement from laser source
0.2 g m^{-2} areal density
10^{-8} spectral normal absorptance at 1.06 μm
70% spectral normal reflectance at 1.06 μm
625 K maximum temperature
0.01 total hemispherical emittance (2-sided, 625 K)
\$0.01 W^{-1} laser cost (k_l)
\$500 m^{-2} optics cost (k_a)
\$50 kWh^{-1} storage cost (k_s)
\$0.1 kWh^{-1} grid energy cost (k_g)
100% grid to storage efficiency (η_{12})
50% storage to laser efficiency (η_{23})
70% transatmospheric propagation efficiency (η_a)
100 operations included in cost minimization (n_o)

System model constants.

Credit: Parkin